Signals & Systems
Quiz 4 Solutions

(a) $\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} -3 & 1 \\ 1 & -3 \end{bmatrix}}_{A} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 1 \\ 1 \end{bmatrix}}_{B} u \qquad \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}_{C} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 0 \end{bmatrix}}_{D} u$

(b) Eigenvalues of $A$: $\det[\lambda I - A] = 0$

$$\det \begin{bmatrix} \lambda+3 & -1 \\ -1 & \lambda+3 \end{bmatrix} = 0$$

$$(\lambda+3)^2 - 1 = \lambda^2 + 6\lambda + 8 = (\lambda+4)(\lambda+2) = 0$$

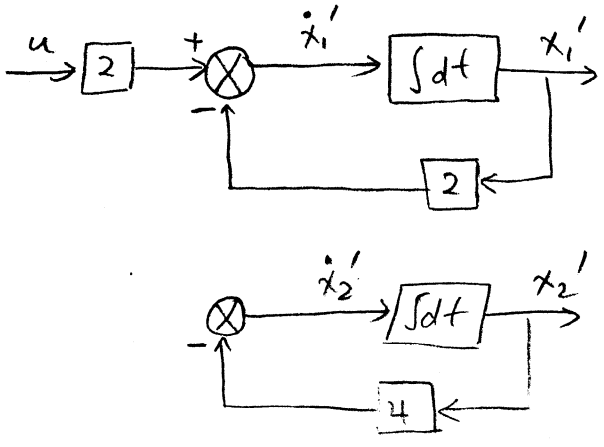
$$\Rightarrow \lambda = -2, -4$$

Two real, negative, distinct eigenvalues $\Rightarrow$ overdamped.

(c) $\vec{x}' = T\vec{x}$, $T = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \Rightarrow T^{-1} = \frac{1}{(-2)} \begin{bmatrix} -1 & -1 \\ -1 & 1 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$

$$B' = TB = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix} \qquad C' = CT^{-1} = \frac{1}{2} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \qquad D' = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

(d)




(e) Block diagram shows clearly that we cannot control state $x_2'$, since the two states are decoupled and $u$ does not affect $x_2'$ $\Rightarrow$ system is uncontrollable